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Spectra of baryons containing two heavy quarks

D.U. Matrasulov[1,a], F.C. Khanna[1,b], Kh.Yu. Rakhimov[2], and Kh.T. Butanov[2]

[1] Physics Department University of Alberta, Edmonton Alberta, T6G 2J1 Canada and TRIUMF, 4004 Wersbrook Mall, V6T2A3 Vancouver, British Columbia, Canada
[2] Heat Physics Department of the Uzbek Academy of Sciences, 28 Katartal St., 700135 Tashkent, Uzbekistan

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Abstract. Doubly heavy baryons, *i.e.*, the baryons containing two heavy quarks are treated in the adiabatic approximation, considering the motion of the light quark as a relativistic motion. The binding energy and mass spectra of doubly heavy baryons are calculated solving the two-center Dirac equation the one-centre Schrödinger equation for Coulomb plus linear potential. The binding energy of the light quark as a function of the distance between heavy quarks is found.

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1 Introduction

The baryons containing two heavy quarks are becoming the subject of extensive theoretical study in recent years. Such an interest can be explained by the forthcoming experiments on the observation of doubly heavy baryons. The succesfull experiments at the Collider Detector at Fermilab Collaboration on the observation of the B_c-meson [1] gives some hope to observing baryons containing two heavy quarks, also. Considerable progress has been made in the understanding of spectroscopy and other properties of doubly heavy baryons since 1989 when the first paper [2] with detailed treatment of such baryons appeared [3–14]. Considerable amount of work is devoted to treatment using QCD approach [11,15]. Important point in the study of the properties of doubly heavy baryons is the calculation of their mass spectra and wave functions. Up to now such calculations have been done using various approaches [2,9,10,12]. In [10] the spectra of baryons containing two heavy quarks is calculated using the nonrelativistic quark model with the potential given by Buchmuller-Tye [16] that divides the QQq system to QQ diqark and the light quark. In this case one obtains central-symmetric two-body potential for the description of three-particle QQq system. Another approach is the calculation of QQq baryon spectra in the framework of the potential model [2–4,12], considering it as a three-particle system. In this case it can be considered as an analogue of the hydrogen molecular ion and one needs to solve the two-center wave equation [12]. Within the potential model approach one should choose a Coulomb plus a confining potential. This method deals with the spectroscopy of the doubly heavy baryons accounting for the relativistic motion of the light quark using the Dirac equation approach. We treat QQq baryon within the Born-Oppenheimer approximation solving first the Dirac equation with two-center Coulomb plus linear potential. After this we solve the Schrödinger equation with central Coulomb plus linear plus the (light quark) energy term to account for the recoil motion of the heavy quarks and calculate the binding energy spectra of the QQq baryons with various quark compositions. Most of the work on the spectroscopy of the doubly heavy baryons does not use the adiabatic approximation, since this leads to additional difficulties with solving two-center wave equations with Coulomb plus confining potential for which variables cannot be separated even in the nonrelativistic case (excluding the two-center Coulomb plus harmonic-oscillator potential). In the case of two-center Dirac equation, variables cannot be separated even for pure Coulomb potential. This fact makes it very difficult to solve the two-center Dirac equation with Coulomb plus confining potential. In this paper we use an approach elaborated for solving relativistic two-center Coulomb problem in our recent paper [17]. This approach is based on the relativistic generalization of the method of linear combination of atomic orbitals (LCAO). For our calculations we use the recently obtained [18] exact ground-state wave functions of the one-center Dirac equation for Coulomb plus linear potential. This allows one to solve the relativistic two-center problem for Coulomb plus lin-

[a] *Permanent address*: Heat Physics Department of the Uzbek Academy of Sciences, 28 Katartal St., 700135, Tashkent, Uzbekistan; e-mail: `dmatrasu@phys.ualberta.ca`
[b] e-mail: `khanna@phys.ualberta.ca`

ear potential with the same success as in the case of two-center problem for pure Coulomb potential [17]. It should be noted that no effects related with spin-orbit or spin-color interaction are considered in this paper. For describing quark-quark interaction we use Coulomb plus linear potential which is a more realistic quark-quark potential (see [19] and references therein). This paper is organized as follows: in sect. 2, an outline of the adiabatic approximation for QQq baryon is given. In sect. 3 the solution of two-center Dirac equation with Coulomb plus linear potential is presented. In sect. 4, the Schrödinger equation describing the recoil motion of heavy quarks is solved to calculate the mass spectra of doubly heavy baryons with various quark compositions.

2 Adiabatic approximation for doubly heavy baryons

The Hamiltonian of QQq system that includes the relativistic motion of the light quark is written as

$$H = H_{\mathrm{q}} + H_{\mathrm{QQ}}, \tag{1}$$

where

$$H_{\mathrm{q}} = \vec{\alpha}\vec{p}_{\mathrm{q}} + \beta(m_{\mathrm{q}} + V_{\mathrm{conf}}(r,R)) + V_{\mathrm{Coul}}(r,R) - 2V_0\,, \tag{2}$$

(the system of units $\hbar = c = 1$ is used) is the Hamiltonian describing the motion of light quark in the field of two heavy quarks. Here $\vec{\alpha}$ and β are the usual Dirac matrices, the confining potential is given by

$$V_{\mathrm{conf}}(r,R) = \lambda(r_1 + r_2),$$

with

$$r_{1,2} = |\vec{r} \pm \vec{R}| = \sqrt{r^2 \pm 2\vec{r}\vec{R} + R^2},$$

where $2R$ is the distance between heavy quarks, and the Coulomb potential energy is

$$V_{\mathrm{Coul}}(r,R) = -\frac{Z}{r_1} - \frac{Z}{r_2},$$

with $r_{1,2} =\mid \vec{r} \pm \vec{R} \mid$ are the distances between light quark and heavy quarks.

Here V_0, λ and $Z = \frac{2}{3}\alpha_s$ are the parameters of the quark-quark interaction potential [12], with Z being the charge of the heavy quarks. Note that the factor 2/3 arises from color matrices.

The Hamiltonian H_{QQ} describes the motion of one heavy quark in the field of another one

$$H_{\mathrm{QQ}} = -\frac{1}{2\bar{M}_{\mathrm{QQ}}}\Delta_R + V(R), \tag{3}$$

with

$$V(R) = -\frac{Z}{R} + \lambda R.$$

In the adiabatic approximation the wave function for doubly heavy baryons is split into light and heavy degrees of freedom and can be represented as

$$\Psi(r,R) = \sum_n \phi_n(R)\Phi_n(r,R)\,.$$

Then QQq system will be described by following two wave equations:

$$\left[\vec{\alpha}\vec{p}_{\mathrm{q}} + \beta(m_{\mathrm{q}} + V_{\mathrm{conf}}(r,R)) + V_{\mathrm{Coul}}(r,R) - \frac{4}{3}V_0\right] \times\Phi(r,R) = E(R)\Phi(r,R) \tag{4}$$

and

$$\left[-\frac{1}{2\bar{M}_{\mathrm{QQ}}}\Delta_R + V_{\mathrm{conf}}(R) + V_{\mathrm{Coul}}(R) + E(R) - \frac{2}{3}V_0\right] \times\phi(R) = \varepsilon\phi(R), \tag{5}$$

where ε is the binding energy of the QQq baryon. Thus, the problem of finding the binding energy spectra of doubly heavy baryon consists of two parts: the problem of solving the two-center Dirac equation and the problem of solving the Schrödinger equation accounting for the recoil motion of heavy quarks. The next section is devoted to the solution of the two-center Dirac equation with Coulomb plus linear potential.

3 Two-center Dirac equation for Coulomb plus linear potential

The motion of the light quark in the field of two heavy quarks is described by the two-center Dirac equation with Coulomb plus linear potential. To find a solution of the two-center Dirac equation is a difficult mathematical problem even in the case of a pure Coulomb potential, since the variables cannot be separated. This makes it impossible to find an exact solution of the two-center Dirac equation. It can be solved only approximately asymptotically or by variational methods. Presently several methods using approximate and variational approaches for solving this equation are available. Recently, the Dirac equation for two-center Coulomb potential was solved by the finite-set basis function method [20,21]. Several methods for aproximate analytical and numerical solution of the two-center Coulomb-Dirac equation have been elaborated by Popov and co-workers [22–24]. The two-center Coulomb-Dirac equation was solved by the relativistic generalization of the LCAO method [17]. In this paper for a solution of the two-center Dirac equation with Coulomb plus linear potential we will use the LCAO method. Thus, the equation we wish to solve is ($m_{\mathrm{q}} = \hbar = c = 1$)

$$H\Psi(r_1,r_2) = \left[\vec{\alpha}\vec{p}_{\mathrm{q}} + \beta(1 + \lambda(r_1+r_2)) - \frac{Z}{r_1} - \frac{Z}{r_2}\right] \times\Psi(r_1,r_2) = E(R)\Psi(r_1,r_2)\,. \tag{6}$$

We will consider the light quark as being in the ground state, since the exact wave functions for corresponding one-center problem available only for this case. For solving it by LCAO method we choose the wave function in the form [17]

$$\Psi = d_1\Psi_1 + d_2\Psi_2\,, \tag{7}$$

where $\Psi_1(\Psi_2)$ is the wave function of the light quark in the field of the first (second) center, *i.e.*, the solution of the one-center Dirac equation for Coulomb plus linear potential

$$d_1 = d_2 = d = \frac{1}{\sqrt{2(1+S)}}, \tag{8}$$

$S = \langle\Psi_1|\Psi_2\rangle = \langle\Psi_2|\Psi_1\rangle$, is the overlap integral.

Then the energy of the light quark can be calculated as

$$E = \langle\Psi \mid H \mid \Psi\rangle, \tag{9}$$

where $\langle\Psi| = [\phi, \chi]$ and $|\Psi\rangle = \left[\begin{smallmatrix}\varphi\\ \chi\end{smallmatrix}\right]$ are defined by (7).

As a solution of the one-center Dirac equation we use a nice result due to Franklin [18], where a simple (but exact) ground-state Dirac wave function for Coulomb plus linear confinement was obtained. It can be written as

$$\varphi_j = Ar_j^{b-1}e^{-ar_j}e^{-\frac{1}{2}\alpha r_j^2}\begin{bmatrix}1\\0\end{bmatrix} \equiv Af_j\begin{bmatrix}1\\0\end{bmatrix}, \tag{10}$$

$$\chi_j = -iA\gamma r_j^{b-1}e^{-ar_j}e^{-\frac{1}{2}\alpha r_j^2}\begin{bmatrix}\cos\theta\\ e^{i\varphi}\sin\theta\end{bmatrix} \equiv -iA\gamma f_j\begin{bmatrix}\cos\theta\\ e^{i\varphi}\sin\theta\end{bmatrix}, \tag{11}$$

where $f_j = r_j^{b-1}e^{-ar_j}e^{-\frac{1}{2}\alpha r_j^2}$ $(j = 1,2)$ is the radial part of the wave function with

$$A = \frac{(2\alpha)^{\frac{3}{4}}}{e^{\frac{a^2}{4\alpha}}}\sqrt{\frac{1-b}{8\pi(2b+1)}\frac{(2\alpha)^{b-1}}{D_{-(2b+1)}(\frac{2a}{\sqrt{2\alpha}})}}$$

being the normalizing constant. The constants a, b and γ are given as

$$a = Q, \quad b = \sqrt{1-Q^2}, \quad \gamma = \frac{Q}{b-1}, \quad \alpha = \sqrt{Q\mu}\,,$$

with $D_{-(2b+1)}$ being the parabolic cylinder function [25]. Note that the functions φ_j, and χ_j, are the solution of the one-center Dirac equation with potential

$$V(r) = -\frac{Q}{r} + \mu r,$$

with parameters $+Q$ and μ different from the ones of the potential in eq. (6) and may be considered as variational parameters. Calculating the energy E, eq. (9), we obtain

$$E(R) = (2\pi A^2R^{2b}c)(1 - 2\pi A^2R^{2b+1}cI_4)^{-1}[bR(I_3+I_4) +2ZbR^2(I_5+I_6)+2\lambda(I_2+I_7)-Z(I_1+I_2)]\,, \tag{12}$$

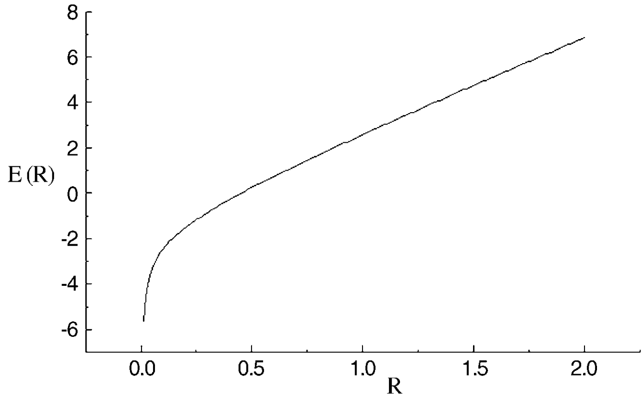

Fig. 1. The binding energy term of the light quark in the field of two heavy quarks calculated using the formula (12). The system of units $m_q = \hbar = c = 1$ is used.

where $c = 2/(b-1)$, and integrals I_1, I_2, ..., I_7 are defined as follows:

$$I_1 = \int_1^\infty\int_{-1}^1 (\xi-\eta)(\xi+\eta)^{2b-2}e^{-R(\xi+\eta)(2a+\alpha R(\xi+\eta))}\,\mathrm{d}\eta\mathrm{d}\xi\,,$$

$$I_2 = \int_1^\infty\int_{-1}^1 \xi(\xi^2-\eta^2)^{b-1}e^{-R(2a\xi+\alpha R(\xi^2+\eta^2))}\,\mathrm{d}\eta\mathrm{d}\xi\,,$$

$$I_3 = \int_1^\infty\int_{-1}^1 (\xi-\eta)(\xi+\eta)^{2b-1}e^{-R(\xi+\eta)(2a+\alpha R(\xi+\eta))}\,\mathrm{d}\eta\mathrm{d}\xi\,,$$

$$I_4 = \int_1^\infty\int_{-1}^1 (\xi^2-\eta^2)^{b}e^{-R(2a\xi+\alpha R(\xi^2+\eta^2))}\,\mathrm{d}\eta\mathrm{d}\xi\,,$$

$$I_5 = \int_1^\infty\int_{-1}^1 \xi(\xi-\eta)(\xi+\eta)^{2b-1}e^{-R(\xi+\eta)(2a+\alpha R(\xi+\eta))}\,\mathrm{d}\eta\mathrm{d}\xi\,,$$

$$I_6 = \int_1^\infty\int_{-1}^1 \xi(\xi^2-\eta^2)^{b}e^{-R(2a\xi+\alpha R(\xi^2+\eta^2))}\,\mathrm{d}\eta\mathrm{d}\xi\,,$$

$$I_7 = \int_1^\infty\int_{-1}^1 \xi(\xi+\eta)^{2b-2}e^{-R(\xi+\eta)(2a+\alpha R(\xi+\eta))}\,\mathrm{d}\eta\mathrm{d}\xi\,.$$

Thus the energy of the light quark in the field of two heavy quarks is expressed by these 7 integrals which, unfortunately, cannot be calculated exactly analytically and must be evaluated numerically. The behaviour of $E(R)$ for small and large R can be estimated analytically. For small R, we have

$$E(R) = \frac{2b}{\frac{\alpha^{b+\frac{1}{2}}}{2\pi A^2c\Gamma(b+\frac{1}{2})}-1}\left[1+\frac{2b\lambda}{\sqrt{\alpha}}\frac{\Gamma(b)}{\Gamma(b+\frac{1}{2})} +\frac{Z(4\alpha)^{b+\frac{1}{2}}}{4b^2(2b+1)\Gamma(b+\frac{1}{2})}R^{2b}+\frac{2\Gamma(b)}{3\Gamma(b+\frac{1}{2})}\sqrt{\alpha}\lambda R^2\right],$$

and, for large R, $E(R)$ is estimated to be

$$E(R) = \frac{2\pi A^2cb}{\alpha^{b+\frac{1}{2}}}\left[2\lambda\Gamma\left(b+\frac{1}{2}\right)R - \Gamma\left(b+\frac{1}{2}\right) -\frac{5\lambda}{\sqrt{\alpha}}\Gamma(b+1)\right].$$

Table 1. The mass spectrum of ccq baryon (in GeV) calculated by solving eq. (13); $m_q = 0.385\,\text{GeV}$ $m_c = 1.486$ GeV n, $\alpha_s = 0.32$, $\lambda = 0.2\,\text{GeV}^2$, $V_0 = -0.1$ GeV, n and l are the principal and orbital quantum numbers of cc diquark, respectively.

n l	0	1	2	3	4	5	6
1	3.241	3.335	3.460	3.617	3.804	4.022	4.271
2	3.346	3.482	3.649	3.848	4.077	4.338	4.629
3	3.492	3.670	3.880	4.120	4.392	4.695	5.028
4	3.681	3.901	4.152	4.435	4.748	5.093	5.469
5	3.911	4.173	4.467	4.791	5.148	5.534	5.952
6	4.184	4.488	4.823	5.189	5.587	6.016	6.476
7	4.498	4.844	5.221	5.629	6.069	6.540	7.042
8	4.854	5.242	5.661	6.111	6.593	7.106	7.650
9	5.252	5.682	6.143	6.635	7.159	7.714	8.300
10	5.692	6.164	6.667	7.201	7.767	8.363	8.991
11	6.174	6.688	7.233	7.809	8.416	9.055	9.725
12	6.698	7.254	7.840	8.458	9.108	9.788	10.500
13	7.264	7.861	8.490	9.150	9.841	10.564	11.318
14	7.872	8.511	9.181	9.883	10.616	11.381	12.179
15	8.521	9.202	9.915	10.658	11.434	12.240	13.078

Table 2. The mass spectrum of bbq baryon (in GeV) calculated by solving eq. (13); $m_q = 0.385$ GeV $m_b = 4.88$ GeV, $\alpha_s = 0.22$, $\lambda = 0.2\,\text{GeV}^2$, $V_0 = -0.1$ GeV, n and l are the principal and orbital quantum numbers of bb diquark, respectively.

n l	0	1	2	3	4	5	6
1	10.158	10.397	10.715	11.112	11.587	12.141	12.774
2	10.424	10.769	11.194	11.698	12.281	12.942	13.682
3	10.796	11.248	11.780	12.390	13.080	13.848	14.695
4	11.275	11.833	12.471	13.188	13.985	14.860	15.814
5	11.860	12.525	13.270	14.093	14.996	15.978	17.039
6	12.551	13.323	14.173	15.104	16.113	17.202	18.370
7	13.349	14.227	15.184	16.221	17.337	18.532	19.807
8	14.253	15.238	16.301	17.444	18.667	19.969	21.350
9	15.264	16.355	17.525	18.774	20.103	21.512	23.000
10	16.381	17.578	18.854	20.210	21.646	23.161	24.755
11	17.605	18.908	20.291	21.753	23.295	24.917	26.617
12	18.934	20.344	21.833	23.402	25.051	26.778	28.586
13	20.370	21.886	23.482	25.157	26.912	28.747	30.661
14	21.913	23.535	25.237	27.019	28.880	30.821	32.842
15	23.562	25.291	27.099	28.987	30.955	33.002	35.129

In fig. 1 the dependence of the energy term $E(R)$ on R, calculated by minimizing $E(R)$ over Q and μ is plotted. The following potential parameters are chosen for these calculations: $\alpha_s = 0.4$, $\lambda = 0.25\,\text{GeV}^2$. They are consistent with the values for $\alpha_s = 0.4$ and $\lambda = 0.25\,\text{GeV}^2$ obtained in other calculations that include a heavy quark [8,19]. It is clear that the binding energy of light quark in the fields of two heavy quarks increases by increasing the distance, R, between heavy quarks. Such a behaviour is to be considered as a consequence of confinement.

4 Spectra of doubly heavy baryons

In this section mass spectra of doubly heavy baryons with various quark compositions are calculated. As was mentioned above the binding-energy spectra of the three-quark system in the adiabatic approximation can be calculated by solving the Schrödinger equation

$$\left[-\frac{1}{2\bar{M}_{QQ}}\Delta_R + V_{\text{conf}}(R) + V_{\text{Coul}}(R) + E(R)\right]\phi_{nl}(R) = \varepsilon_{nl}\phi_{nl}(R), \tag{13}$$

where $E(R)$ is defined by (12) n and l are the principal and orbital quantum numbers QQ system.

Solving this equation numerically we obtain the binding energy spectra of QQq system. Then the mass spectra can be calculated by the formula

$$M_{nl} = M_{QQ} + m_q + \varepsilon_{nl}\,. \tag{14}$$

In tables 1, 2 and 3 the mass spectra of ccq, bbq and bcq baryons are calculated solving eq. (13) numerically and then using formula (14) are presented, respectively. In our calculations we use different values α_s for cc, bb and bc potentials. The values of the constants α_s, λ, and V_0 for the potential

$$V(r) = \frac{2}{3}\left[-\frac{\alpha_s}{R} + \lambda R + V_0\right],$$

Table 3. The mass spectrum of bcq baryon (in GeV) calculated by solving eq. (13); $m_{\rm q} = 0.385$ GeV $m_{\rm c} = 1.486$ GeV and $m_{\rm b} = 4.88$ GeV, $\alpha_s = 0.30$, $\lambda = 0.2\,{\rm GeV}^2$, $V_0 = -0.1$ GeV, n and l are the principal and orbital quantum numbers of bc diquark, respectively.

n l	0	1	2	3	4	5	6
1	6.693	6.852	7.064	7.329	7.646	8.015	8.437
2	6.870	7.101	7.384	7.720	8.108	8.549	9.042
3	7.118	7.420	7.774	8.181	8.641	9.153	9.718
4	7.438	7.810	8.235	8.713	9.244	9.828	10.464
5	7.828	8.271	8.767	9.316	9.918	10.573	11.280
6	8.289	8.803	9.370	9.990	10.663	11.389	12.168
7	8.820	9.406	10.044	10.735	11.479	12.276	13.126
8	9.423	10.079	10.788	11.551	12.366	13.234	14.154
9	10.097	10.824	11.604	12.437	13.323	14.262	15.254
10	10.842	11.640	12.491	13.395	14.352	15.362	16.425
11	11.657	12.526	13.448	14.423	15.451	16.532	17.666
12	12.544	13.484	14.476	15.522	16.621	17.773	18.978
13	13.501	14.512	15.576	16.693	17.863	19.085	20.361
14	14.530	15.611	16.746	17.934	19.175	20.469	21.815
15	15.629	16.781	17.987	19.246	20.558	21.922	23.340

are given in each table. Again the choice of parameters is consistent with the values previously chosen by other authors [8]. It is possible that for cc and bb systems, the parameters will change somewhat but it is expected that such an effect will be small. As is seen from these tables, our calculations are in good accordance with the results of [10]. Note that this potential does not include any terms, describing spin-orbit, spin-color interactions.

5 Conclusion

In this work the doubly heavy baryon is treated in the adiabatic approximation, with the light quark being in a relativistic motion. This required solving

a) a two-center Dirac equation with Coulomb plus linear potential and
b) the Schrödinger equation with Coulomb plus linear potential plus the energy term of light quark in the field of two heavy quarks.

The mass spectra are displayed for the case of $n_{\rm q} = 1$ and $l_{\rm q} = 0$ (where $n_{\rm q} = 1$ and $l_{\rm q} = 0$ are the principal and orbital quantum numbers of the light quark) and using a specific set of potential parameters that are consistent with previous attempts of calculations with two heavy quarks. The variations of these parameters will change the spectra, but it is anticipated that changes in α_s for bbq, bcq and ccq will be small. It should be emphasized again that the spin-spin and spin-orbit interactions would define the baryon spectra that can be compared with experiment. The present work may be viewed as an attempt to explore the spectra using the Dirac equation compared to working with the Schrödinger equation for the light quark. More realistic calculations that include the role of spin and color effects are needed. It is to be hoped that experiments at Fermilab and CERN would undertake such a study to help establish the technique discussed here for obtaining a realistic baryon spectrum. In such a case a careful study with a variation of the parameters α_s and λ need to be carried out to understand QCD of small energies and momenta.

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